SUBJECT and GRADE	Mathematics Grade 10	
TERM 1	Week 3: Algebraic Expressions	
TOPIC	Algebraic Fractions	
AIMS OF LESSON	To simplify Algebraic fractions	
RESOURCES	***Paper based resources***	***Digital resources***
	Please refer to the chapter in your textbook on Algebraic Fractions.	*https://www.youtube.com/watch?v=OiBN-Xd2wek* *https://www.youtube.com/watch?v=lBQmy1IMko8* *https://www.youtube.com/watch?v=2SSGrCzT4Y0* *https://www.youtube.com/watch?v=uO-hCeFPXjo*
INTRODUCTION	In this lesson we will look at the simplification of Algebraic Fractions. It latches on work on fractions from previous grades and forms an important basis for further study in Mathematics.	
CONCEPTS/ SKILLS	• simplify fractions by cancelling • simplify fractions by factorising first • multiplication/ division of algebraic fractions • addition/ subtraction of algebraic fractions using an LCM (lowest common multiple)	
Lesson 1	**Revision of Gr 9 Algebraic Fractions**	

Simplifying fractions:

Example 1: $\frac{24}{36}=\frac{\cancel{12}\times 2}{\cancel{12}\times 3}=\frac{2}{3}$

Example 2: $\frac{y^2}{y^3}=\frac{\cancel{y}\times\cancel{y}}{\cancel{y}\times\cancel{y}\times y}=\frac{1}{y}$

NB!
We can ONLY divide (cancel) the **same term/ same bracket** if the numerator AND denominator is ONE term (product expression) or they consist of the same term(s)

Example 3: $\frac{2(x+3)^2}{10(x+3)}=\frac{\cancel{2}\cancel{(x+3)}(x+3)}{\cancel{2}\times 5\cancel{(x+3)}}=\frac{x+3}{5}$

Example 4: $\frac{\cancel{2x+3}}{\cancel{2x+3}}=1$

Example 5: $\frac{2x+3}{x+3}$

We CANNOT cancel the **3** since the numerator and denominator is not ONE TERM and/or the terms are not the same; Answer remains $\frac{2x+3}{x+3}$

CAN YOU?

Simplify the following expressions:

1. $\frac{48}{42}$
2. $\frac{x^2ya^3}{xy}$
3. $\frac{2x(2x-1)}{6x^2(2x-1)^3}$
4. $\frac{x^2-x+3}{x^2-x+3}$
5. $\frac{3x-2}{x-2}$
6. $\frac{36a^2b^5}{6ab}$
7. $\frac{63xy^2}{28x^3}$
8. $\frac{3\left(-2x^2y\right)^3(xy^3)^2}{12x^7y^{10}}$

Example 6: Simplify: $\frac{12x^4y}{20xy^3} = \frac{4\times3x.x^3y}{4\times5xy.y^2}$

You do not need to show this step!!

$= \frac{3x^3}{5y^2}$

Example 7: $\frac{2(3x)^2(-xy)^3}{6x^5y^4} = \frac{2(3^2x^2)(-x^3y^3)}{6x^5y^4}$

Remove brackets by applying exponent laws

$\frac{-18x^5y^3}{6x^5y^4} = -\frac{3}{y}$

Answers:

1. $\frac{8}{7}$
2. xa^3
3. $\frac{1}{3x(2x-1)^2}$
4. 1
5. $\frac{3x-2}{x-2}$
6. $6ab^4$
7. $\frac{9y^2}{4x^2}$
8. $\frac{-2x}{y}$

Lesson 2	Grade 10 Algebraic Fractions where numerators and/ or denominators must be factorised

Simplify the following:

Example 1: $\frac{4x+6}{2}$

We cannot divide with 2 since the numerator is not a monomial (one term). To make a polynomial (sum expression) a monomial (product expression) we must **FACTORISE**

$= \frac{2(2x+3)}{2}$

$= 2x + 3$

Example 2: $\frac{3a^2+6a}{6a}$

We cannot divide with $6a$ since the numerator is not a monomial (one term). Hence, factorise numerator first

$= \frac{3a(a+2)}{6a} = \frac{a+2}{2}$

Example 3: $\frac{2x^2-8}{x-2} = \frac{2(x^2-4)}{x-2}$

Remember? 1st look for Common factor. If 2 terms $\Rightarrow$ look for difference of 2 squares, etc.

$= \frac{2(x+2)(x-2)}{(x-2)}$

NOTE: We can put $\boldsymbol{x-2}$ in a bracket, since there are NO other terms in the denominator.

$= 2(x+2)$

Example 3: $\frac{-18x-36}{6x^2-6x-36}$

$= \frac{-18(x+2)}{6(x^2-x-6)} = \frac{-18(x+2)}{6(x-3)(x+2)} = \frac{-3}{x-3}$

Again: We cannot cancel the -3 since the denominator is not 1 term; it is thus in its simplest form.

CAN YOU?

Simplify:

1. $\frac{2m-4n+6p}{2}$
2. $\frac{x^2-2x}{2x}$
3. $\frac{(x-2)(3x-2)}{(x-2)}$
4. $\frac{4x^3+4x^2+5x+5}{4x^2+5}$
5. $\frac{(a+b)^2}{a^2+ab}$
6. $\frac{a^2+a}{a^2+2a+1}$
7. $\frac{mx-nx}{ny-my}$
8. $\frac{3x^2-8x-3}{6x^2+2x}$
9. $\frac{p^3+xp^2-y^2p-xy^2}{p^2+py+px+xy}$
10. $\frac{x^2-4}{x^2-4x+4}$
11. $\frac{3x^2-6x+12}{x^3+8}$
12. $\frac{x^3-y^3}{x^2-y^2}$
13. $\frac{x^2-2x+1}{2x}$

Answers:

1. $m-2n+3p$
2. $\frac{x-2}{2}$
3. $3x-2$
4. $x+1$
5. $\frac{a+b}{a}$
6. $\frac{a}{a+1}$
7. $\frac{-x}{y}$
8. $\frac{x-3}{2x}$
9. $p-y$
10. $\frac{x+2}{x-2}$
11. $\frac{3}{x+2}$
12. $\frac{(x^2+xy+y^2)}{(x+y)}$
13. $\frac{x}{2}-1+\frac{1}{2x}$

Lesson 3	Multiplication and Division of Algebraic Fractions

Rules:

1. $\frac{a}{b} \times \frac{c}{d} = \frac{ac}{bd}$; $b, d \neq 0$
2. $\frac{a}{b} \div \frac{c}{d} = \frac{a}{b} \times \frac{\mathbf{d}}{\mathbf{c}}$

Impact of $\times$ and $\div$:

1. $p \times q \div z = \frac{p}{1} \times \frac{q}{1} \times \frac{1}{z}$
2. $p \div q \times z = \frac{p}{1} \times \frac{1}{q} \times \frac{z}{1}$
3. $p \div q \div z = \frac{p}{1} \times \frac{1}{q} \times \frac{1}{z}$
4. $p \div (q+z) = p \times \frac{1}{(q+z)}$
5. $\frac{-a}{b} = \frac{a}{-b} = -\frac{a}{b}$
6. $b - a = -(a - b)$

Simplify:

Example 1: $\frac{a+b}{a-b} \times \frac{b-a}{a}$

$= \frac{(a+b)}{(a-b)} \times \frac{-(a-b)}{a}$

$= \frac{-(a+b)(a-b)}{a(a-b)}$

$= -\frac{(a+b)}{a}$

Rule 1: numerator × numerator and denominator × denominator

Example 2: $\frac{2a^2-ab}{5} \times \frac{15}{(2a-b)}$

$= \frac{a(2a-b)}{5} \times \frac{15}{(2a-b)}$

$= \frac{15a(2a-b)}{5(2a-b)} = 3a$

× and ÷ combine terms into a monomial, hence we can cancel the same term/ bracket in the numerator and denominator

Example 3: $\frac{p^2-2p}{4} \div \frac{pq-2q}{8}$

$= \frac{p^2-2p}{4} \times \frac{\mathbf{8}}{\mathbf{pq-2q}}$

$= \frac{8(p^2-2p)}{4(pq-2q)} = \frac{8p(p-2)}{4q(p-2)} = \frac{2p}{q}$

Rule 2: If we ÷ , we × the dividend with the ***multiplicative inverse*** of the divisor, called the ***reciprocal***.

Example 4: $\frac{3x^2+3x}{8x^3+27} \div \frac{x+1}{2x^2+x-3} \times \frac{4x^2-6x+9}{x-1}$

$= \frac{3x^2+3x}{8x^3+27} \times \frac{\mathbf{2x^2+x-3}}{\mathbf{x+1}} \times \frac{4x^2-6x+9}{x-1}$

$= \frac{3x(x+1)}{(2x+3)(4x^2-6x+9)} \times \frac{(2x+3)(x-1)}{(x+1)} \times \frac{(4x^2-6x+9)}{(x-1)}$

$= 3x$

CAN YOU?

Simplify:

1. $\frac{4a}{3xy} \div \frac{2a}{6x^2}$
2. $\frac{4ay}{3xy} \times \frac{2by}{6y^2} \div \frac{5ax}{9x^2y}$
3. $\frac{3(a-2)}{(a+2)} \div \frac{4a-8}{a^2-4}$
4. $\frac{2a-4+ab-2b}{ab+2a} \times \frac{1}{a^2-4}$
5. $\frac{x^2-3x}{4-x^2} \times \frac{x^2-2x}{x^2-2x-3}$
6. $\frac{p^2-4}{2p+1} \div \frac{p^2-2p}{2p^2-5p-3}$
7. $\frac{y^3-1}{3+2y-y^2} \times \frac{y^2-1}{y^2-2y+1}$
8. $\frac{b^2-a^2}{a^2-ab-2b^2} \div \frac{a^2-3ab+2b^2}{a^2-4ab+4b^2}$
9. $\frac{2x}{2x-6} \div \frac{2x^2+4x}{2x^2-18} \times \frac{x^2+4x+4}{x^2+5x+6}$

Answers:

1. $\frac{4x}{y}$
2. $\frac{b}{5}$
3. $\frac{3(a-2)}{4}$
4. $\frac{1}{a(a+2)}$
5. $\frac{-x^2}{(x+2)(x+1)}$
6. $(p+2)(p-3)$
7. $\frac{-(y^2+y+1)}{(y-3)}$
8. -1
9. 1

the **dividend** (the number to divide into)

the **divisor** (the number with which we divide)

NOTE: $6x^2 \div 2x = 3x$

the **quotient** (the answer when we divide)

Lesson 4 + 5	Addition and Subtraction of Algebraic Fractions

Rules:

- $\frac{a}{b}+\frac{c}{b}=\frac{a+c}{b}$

Example 1: $\frac{2}{5}+\frac{1}{5}=\frac{1+2}{5}=\frac{3}{5}$

> If the denominators are the same, we add the numerators (denominator remains the same)

Example 2: $\frac{2x}{7y}-\frac{3}{7y}=\frac{2x-3}{7y}$

- $\frac{a}{b}+\frac{c}{d}=\frac{a(d)}{bd}+\frac{c(b)}{bd}=\frac{a(d)+c(b)}{bd}$

> If the denominators different, we can make them the same by using the LCM (**L**owest **C**ommon **M**ultiple)

LCM: Smallest number into which **each** denominator can divide.

Determine the LCM of:

> Break up numbers into factors.
> LCM: product of highest power of each **different** factor

- 2, 6 and 8
  $\therefore$ 2, (2×3) and $(2\times 4)\Rightarrow$ LCM: $2\times 3\times 4=\mathbf{24}$
- $2, 4$ and 6 $\therefore 1, 2^2$ and $2\times 3\Rightarrow$ LCM: $1\times 2^2\times 3=\mathbf{12}$
- 5 and 7$\Rightarrow$ LCM: $5\times 7=\mathbf{35}$
- x, x^2 and $x^3\Rightarrow$ LCM is the **highest power** of x $\therefore$ $\boldsymbol{x^3}$
- $2x, (x-4)$ and $3\Rightarrow$ LCM: $\mathbf{6x(x-4)}$
- $(x^2+1), (x+1)$ and $(x+1)^2\Rightarrow$ LCM: $(x^2+1)(x+1)^2$
- $x^2-x, (x-1)^2$ and x^2 : **factorise terms first**:
  $\therefore x(x-1), (x-1)^2$ and $x^2\Rightarrow$ LCM: $x^2(x-1)^2$
- $(x+2)$ and $(x-1)\Rightarrow$ LCM: $(x+2)(x-1)$

> **NOTE**: $x^2+1\neq(x+1)^2$ they are thus different factors

Equivalent fractions: Fractions with the same value but denominators: e.g. $\frac{1}{2}=\frac{3}{6}=\frac{7}{14}$, etc.

Example 3: $\frac{\mathbf{2}}{\mathbf{3}}+\frac{5}{6}=\frac{\mathbf{4}}{\mathbf{6}}+\frac{5}{6}$
$=\frac{9}{6}$

> LCM of **3** and **6** is **6**. Hence we change $\frac{2}{3}$ to its equivalent fraction with denominator **6**, which is $\frac{4}{6}$: $\left[\frac{2\times(\mathbf{2})}{3\times(\mathbf{2})}\right]$

Example 4: $\frac{2}{a^2b}-\frac{3}{ab}+\frac{4}{a}$

$=\frac{2(1)-3(a)+4(ab)}{a^2b}$

$=\frac{2-3a+4ab}{a^2b}$

> LCM: a^2b.
> Divide each denominator into the LCM and $\times$ with numerator. Write fraction with LCM as common denominator

CAN YOU?
Simplify

1. $\frac{x+y}{x}-\frac{y^2+x^2}{x^2}$
2. $\frac{3a-1}{2a^2}+\frac{2}{3a}-\frac{a-3}{3a^2}$
3. $\frac{3}{(x+5)}-\frac{3}{(x+3)}$
4. $\frac{3}{(x+1)}-\frac{2x}{(x-1)^2}$
5. $\frac{3}{(x+2)}+\frac{2}{(1-2x)}$
6. $\frac{2}{(x^2-7x+12)}-\frac{1}{(x^2-4x+3)}$
7. $\frac{a-2}{a^2+4a+4}+\frac{3}{a+2}$
8. $\frac{b}{b+3}-\frac{6b}{9-b^2}$
9. $\frac{2}{(x^2+6x+8)}-\frac{1}{(x^2+5x+6)}$
10. $\frac{2a+1}{3a^3+24}-\frac{4}{3a+6}+\frac{1}{2a^2-4a+8}$

Example 5: $\frac{3}{(x+1)^2} - \frac{2}{3x+3}$

$$= \frac{3}{(x+1)^2} - \frac{2}{3(x+1)}$$

Factorise denominators first

$$= \frac{3(3)-2(x+1)}{3(x+1)^2}$$

LCM: $3(x+1)^2$

$$= \frac{9-2x-2}{3(x+1)^2} = \frac{7-2x}{3(x+1)^2}$$

Example 6: $\frac{p}{p^2-16} - \frac{p+1}{p^2-3p-4}$

$$= \frac{p}{(p+4)(p-4)} - \frac{p+1}{(p-4)(p+1)}$$

Factorise denominators first

$$= \frac{p(p+1)-(p+1)(p+4)}{(p+1)(p-4)(p+4)} = \frac{p^2+p-(p^2+5p+4)}{(p+1)(p-4)(p+4)} = \frac{p^2+p-p^2-5p-4}{(p+1)(p-4)(p+4)}$$

$$= \frac{-4p-4}{(p+1)(p-4)(p+4)} = \frac{-4(p+1)}{(p+1)(p-4)(p+4)} = \frac{-4}{(p-4)(p+4)}$$

Answers:

1. $\frac{y(x-y)}{x^2}$
2. $\frac{5a-3}{2a^2}$
3. $\frac{-6}{(x-5)(x+3)}$
4. $\frac{x^2-8x+3}{(x+1)(x-1)^2}$
5. $\frac{(4x-7)}{(x+2)(2x-1)}$
6. $\frac{(x+2)}{(x-4)(x-3)(x-1)}$
7. $\frac{4(a+1)}{(a+2)^2}$
8. $\frac{b}{b-3}$
9. $\frac{1}{(x+4)(x+3)}$
10. $\frac{-8a^2+25a-24}{6(a+2)(a^2-2a+4)}$

Mixed Exercise on Algebraic Fractions

Revision Exercise

1. $1\frac{-16x^2y^3}{4x^3y^2}$
2. $\frac{3x^3-2x^2-5x}{x^2+x}$
3. $\frac{3x^2-2x-1}{x^2-1}$
4. $\frac{(x-5)^2}{x^2-2x-15}$
5. $\frac{ax+ay-cx-cy}{ax+ay+cx+cy}$
6. $\frac{(x-5)^2}{x^2-2x-15} \times \frac{3x^2+24x+45}{x^2-25} \div \frac{3x^2-9x}{x^2-9}$
7. $\frac{x+3}{x^2-1} - \frac{3}{x^2+2x+1}$
8. $\frac{2}{x-1} + \frac{3}{1-x} + \frac{2}{x^2-1}$

Revision Exercise Answers:

1. $\frac{-4y^2}{x}$
2. $3x-5$
3. $\frac{3x+1}{x+1}$
4. $\frac{x-5}{x+3}$
5. $\frac{a-c}{a+c}$
6. $\frac{x+3}{x}$
7. $\frac{x^2+x+6}{(x+1)^2(x-1)}$
8. $\frac{-1}{x+1}$

ACTIVITIES	*Consider other exercises from your Mathematics Textbook*
CONSOLIDATION	• We can ONLY divide (cancel) the **same term/ same bracket** if the numerator AND denominator is ONE term (product expression) or they consist of the same term(s); if it is not, FACTORISE numerator and denominator • $\frac{-a}{b} = \frac{a}{-b} = -\frac{a}{b}$ • Rules for × and ÷ of algebraic fractions: ○ $\frac{a}{b} \times \frac{c}{d} = \frac{ac}{bd}$; $b, d \neq 0$ ○ $\frac{a}{b} \div \frac{\boldsymbol{c}}{\boldsymbol{d}} = \frac{a}{b} \times \frac{\boldsymbol{d}}{\boldsymbol{c}}$ • Rules for + and – of algebraic fractions: ○ factorise first and find the LCM of the denominators ○ $\frac{a}{b} + \frac{c}{b} = \frac{a+c}{b}$ ○ if the denominators are different, we can make them the same by using the LCM (Lowest Common Multiple)
VALUES	*Dear learner. Hope you are still practicing Mathematics every day. Hang in there. Your HARD WORK will REAP SUCCESS.*